1. Consider the disturbance given by the expression
$\vec{E}(z,t) = \left[\hat{i}\cos(\omega t) + \hat{j}\cos(\omega t - \pi/2)\right]E_0\sin(kz)$, where $\hat{i}$ and $\hat{j}$ are unit vectors in the x- and y-directions. What kind of wave is it? Draw a sketch showing its main features.

Solution:

First, since the z-dependence is alone inside the sine function we see that this is a standing wave. Then we notice that light is equal split between $\hat{x}$ and $\hat{y}$, with a $\pi/2$ phase shift, implying that the light is circularly polarized.
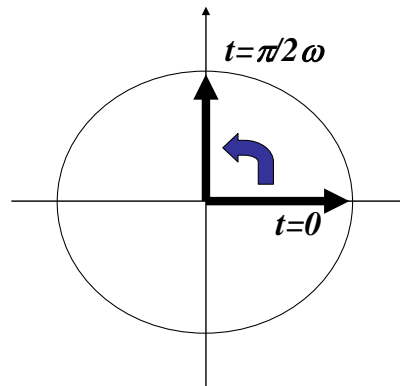
2. Substances such as sugar, Karo syrup, and insulin are optically active; they rotate the plane of polarization in proportion to both the path length and the concentration of the solution. A glass vessel is placed between two perfect crossed linear polarizers, and 50% of the natural light incident on the first polarizer is transmitted through the second polarizer. By how much did the sugar solution in the cell rotate the light that passed by the first polarizer?

Solution:

Consider initially unpolarized light of intensity I_0. After transmission through the first polarizer only half the light intensity is left. Since the observers sees $I_0/2$, we must get all the remaining light through the last polarizer. That means the poloarized light must be in the same direction as the second polarizer. Since the polarizers are crossed we must rotate the light by 90 degrees.

3. Pedrotti[3], 3rd edition, problem 15-1

Solution:

Consider initially unpolarized light of intensity I_0. The intensity after the first polarizer is given by $I_1 = I_0/2$. Through the second polarizer $I_2 = I_1 \cos^2 30 = \dfrac{I_0}{2}\cos^2 30$. Finally through the last polarizer

$$I_3 = I_2 \cos^2(60-30) = I_1 \cos^2(60-30)\cos^2 30 = \frac{I_0}{2}\cos^2 30 \cos^2(60-30) = 0.2815 I_0$$

Thus 28.15% of the intensity is transmitted.

4. Pedrotti[3], 3rd edition, problem 15-2

Solution:

Reflected light is linerly polarized at the Brewster's angle: $\tan\theta_B = \dfrac{n_2}{n_1}$. *Thus for*

Internal Reflection we have $\theta_B = \tan^{-1}\dfrac{n_{air}}{n_{diam}} = \tan^{-1}\dfrac{1}{2.42} = 22.5°$. *And for External*

Reflection $\theta_B = \tan^{-1}\dfrac{n_{diam}}{n_{air}} = \tan^{-1}\dfrac{2.42}{1} = 67.5°$.

5. An ideal polarizer is rotated at a rate ω between a similar pair of stationary crossed polarizers. Show that the emergent flux density will be modulated at four times the rotational frequency. In other words, show that $I = \dfrac{I_1}{8}(1-\cos(4\omega t))$, where I_1 is the flux density emerging from the first polarizer and I is the final flux density.

Solution:

Let $\theta = \omega t$. The emergent light intensity is given by Malus's Law:

$$I = I_0 \cos^2\theta \cos^2(90-\theta) = I_0 \cos^2\theta \sin^2(\theta) = \frac{1}{4}I_0(1-\cos 2\theta)(1+\cos 2\theta)$$

$$= \frac{1}{4}I_0(1-\cos^2 2\theta) = \frac{1}{8}I_0(1-\cos 4\theta)$$

6. Pedrotti[3], 3rd edition, problem 14-2

Solution:

a) $\dfrac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$, *linearly polarized at -45 degrees*

b) $\dfrac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, *linearly polarized at 45 degrees*

c) $\dfrac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \dfrac{1}{\sqrt{2}}(1-i) \end{bmatrix}$, *right elliptically polarized at 45 degrees*

d) $\dfrac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$, *left circularly polarized*